Heavy Quark-Antiquark Potential from QCD and Quarkonium Spectra ◇

A. Laschka, N. Kaiser, and W. Weise

The quarkonium potential has been studied by lattice simulations as well as in perturbative QCD. It is an ideal object for exploring the interplay between perturbative and non-perturbative strong interaction physics. However, QCD perturbation theory tends to fail already at very small distances. This behaviour is usually improved by subtracting the leading renormalon pole [1]. We use a different definition of the r-space potential that will be explained in the following.

The static, quark mass independent potential in momentum space (given here at two-loop order) [2],

$$\tilde{V}^{(0)}(q) = -\frac{4\pi C_F \alpha_s(q)}{q^2}\left\{1 + \frac{\alpha_s(q)}{4\pi}\, a_1 + \left(\frac{\alpha_s(q)}{4\pi}\right)^2 a_2\right\},$$

is no more valid for $q \lesssim 1$ GeV since α_s becomes too large. This excludes a standard Fourier transformation to momentum space. Usually, this problem is avoided by an expansion of the running coupling $\alpha_s(q)$ in a power series about a fixed scale μ. Our alternative definition of the coordinate space potential is based on a restricted numerical Fourier transform with a low-momentum cutoff q_{min}:

$$V^{(0)}(|\vec{r}|) = \int\limits_{|\vec{q}|>q_{\mathrm{min}}} \frac{d^3q}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}}\, \tilde{V}^{(0)}(|\vec{q}|).$$

The full RGE running of $\alpha_s(q)$ is included in this construction without any expansion. The resulting (perturbative) potential depends only weakly on the cutoff q_{min}, apart from an overall additive constant, and can be matched at distances r between 0.1 fm and 0.2 fm to the static potential obtained from lattice QCD [3] (Fig. 1).

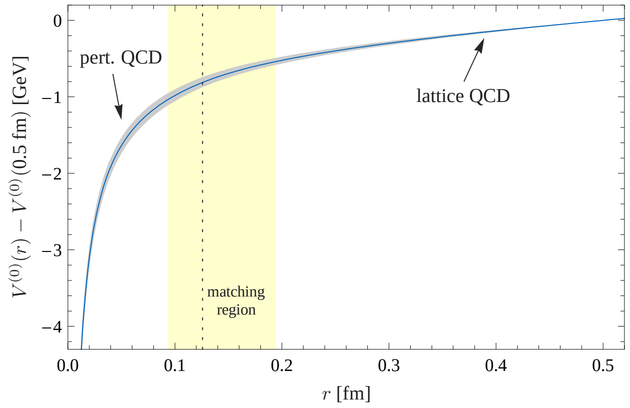

Fig. 1: Smooth (static) $q\bar{q}$ potential in coordinate space, derived from QCD. It is applicable to distances up to $r \sim 1$ fm.

The error band of the curve reflects uncertainties in the Sommer scale $r_0 = 0.50 \pm 0.03$ fm (length scale on the lattice) and uncertainties in the value of $\alpha_s(m_Z) = 0.1176 \pm 0.0020$.

A non-perturbative expression in chromoelectric field correlators has been determined to extract the quark-antiquark potential at order $1/m$ in the heavy quark mass [4]. Recent lattice computations at order $1/m$ [5] can be matched analogously at distances $r \approx 0.1$ fm to the perturbative potential. This allows for an analysis of the influence of the order $1/m$ potential on quarkonium spectra.

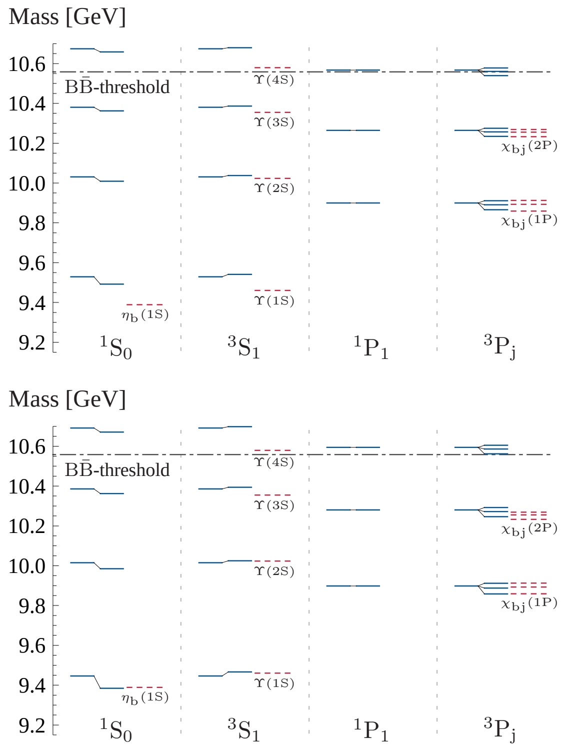

Fig. 2: Bottomonium spectrum: different potentials $V = V^{(0)}$ (top) and $V = V^{(0)} + \frac{V^{(1)}}{m/2}$ (bottom) are used as input. The dashed lines represent the experimentally observed mass values.

As shown in Fig. 2, the experimental bottomonium spectrum can be well reproduced if the $1/m$ potential is included. The spin dependent splittings are obtained from one-gluon exchange with an adjusted $\alpha_s = 0.31$. The next step is the systematic inclusion of order $1/m^2$-effects (spin-dependent terms) within the same framework. This will hopefully lead to an improvement in the description of the charmonium spectrum, where $1/m^2$-effects are expected to be sizeable.

References

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