Scaling Analysis of Two-Flavor Lattice QCD Results ◇

B. Klein and J. Braun [a]

[a] TRIUMF, Vancouver, BC

The nature of the QCD phase transition is a topic which is still under intense discussion. Experimentally, results from RHIC at BNL and from CERN have improved our knowledge of the transition and revealed surprising properties of the high-temperature state, and future results from the LHC at CERN and FAIR at GSI are eagerly awaited and will require extensive theoretical analysis. While non-perturbative methods such as Dyson-Schwinger Equation approaches [1] and functional Renormalization Group methods are now also applied to QCD thermodynamics [2,3], most of our theoretical knowledge comes from phenomenological models such as the Polyakov-loop extended Nambu–Jona-Lasinio model, and primarily from simulations of QCD on the lattice.

Lattice simulations have made significant progress in the exploration of the thermodynamics of QCD. In particular with regard to the physically most relevant situation with two light and one heavy (strange) quark flavors, it now appears very likely that the transition is a continuous crossover [4]. The ratio of the heavy and light quark masses in such simulations has approached realistic values, such that the effects of light Goldstone modes appear to become observable in the results [5].

Nevertheless, some limitations of these simulations remain: even now, the masses of the lightest Goldstone-modes are large, the chiral symmetry of QCD is explicitly broken by the quark masses, and the simulations always necessarily take place in a finite simulation volume, which precludes an actual phase transition. For these reasons, an analysis of the transition requires the consideration of scaling and finite-size scaling effects to determine the order of the transition without doubt and to identify a possible universality class. This would allow to deduce the underlying degrees of freedom and the relevant dynamics.

In particular, a controversy remains with regard to the order of the phase transition with two light flavors of quarks in the so-called *staggered* realization of quarks in lattice simulations. A careful finite-size scaling analysis [6] reveals no scaling behavior consistent with the expected O(4) or O(2) universality class. It is important to settle this question, since the same quark representation is also widely used for the physically more realistic case of three quark flavors.

We compare the lattice simulation results to the scaling and finite-size scaling functions for the expected O(4) universality class, which we obtained with a functional Renormalization Group calculation [7,8]. Our results with regard to finite-size scaling suggest that the lattice results are not in a region where one expects to see large effects. Assuming infinite-volume behavior, our results suggest that scaling behavior could be masked by large corrections due

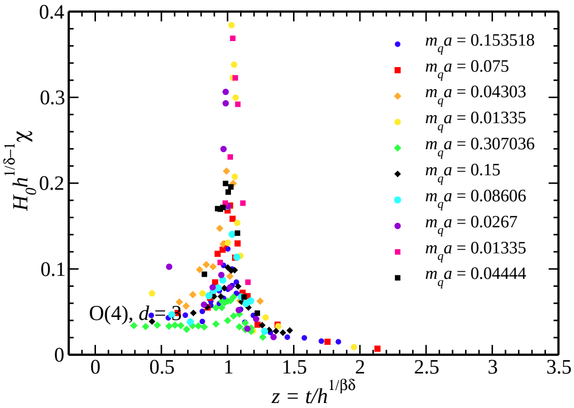

Fig. 1: Results for the chiral susceptibility as a function of the scaling variable $z = t/h^{1/(\beta\delta)}$ from [6]. The data used in the analysis is taken from [6], while the plot is a result of our own scaling analysis which assumes O(4) $d = 3$ scaling. The peak heights clearly are not the same, as would be expected in the scaling regime. Either the scaling is indeed not in the expected universality class, or scaling corrections are too large for current quark mass values.

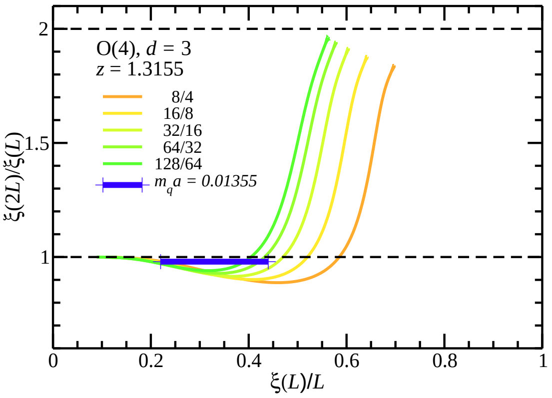

Fig. 2: Ratio of correlation length ξ for volume sizes $L = 2 \times 16$ and $L = 16$ as a function of the ratio between correlation length and volume size ξ/L for the smaller volume. The blue bar represents the result from [6]. The absolute value for ξ/L is estimated from the susceptibility and the mass of the pion as the lightest mode. The comparison with our results for the scaling functions shows that no large finite-size scaling effects can be expected.

References

[1] C. S. Fischer, J.Phys. **G32** (2006) R253
[2] J. Braun, arXiv:0810.1727 [hep-ph].
[3] J. Braun, H. Gies, J. M. Pawlowski, arXiv:0708.2413 [hep-th].
[4] Y. Aoki *et al.*, Nature **443** (2006) 675
[5] F. Karsch *et al.*, Nucl. Phys. **A820** (2009) 99C
[6] M. D'Elia, A. Di Giacomo and C. Pica, Phys. Rev. **D72** (2005) 114510
[7] J. Braun and B. Klein, Phys. Rev. **D77** (2008) 096008
[8] J. Braun and B. Klein, arXiv:0810.0857 [hep-ph].

◇ Work supported in part by BMBF and by the DFG excellence cluster "Origin and Structure of the Universe".