Vacuum Decay Time in Strong Electromagnetic Fields ◇

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The vacuum state of quantum electrodynamics (QED) is metastable in the presence of electrical fields of any strength, as exhibited by the nonzero imaginary part of the Euler-Heisenberg-Schwinger (EHS) effective action [1]. The total decay probability is suppressed exponentially by the critical strength $E_0 = m^2c^3/e\hbar$, corresponding to a potential step equal to the rest mass of the electron. High intensity short pulse laser technology has developed rapidly recently and laboratory experiments near the critical field are conceivable in the near future [2]. However, destablizing the vacuum with laser pulses requires decay dynamics on the time scale of the pulse length. We therefore investigate the time scale at which non-perturbative effects of QED may become observable [3].

With the assumptions of adiabatic switching and constant, homogeneous field, the EHS effective action may be used to calculate Schwinger-mechanism pair creation as a tunneling probability, $\Gamma(\epsilon_\perp^2) = \exp\left[-\frac{\pi}{eE}\epsilon_\perp^2\right]$. To determine the rate at which the field is converted into pairs, we weight the probability by the transverse energy of the created pair and integrate over transverse momentum, giving

$$\frac{d\langle u_m\rangle}{dt} = \omega_0 E^2 e^{\frac{-\pi E_0}{E}}\left\{1 + h\left(\sqrt{\frac{\pi E_0}{E}}\right)\right\}, \tag{1}$$

in which $\omega_0 := \alpha mc^2/\pi^2\hbar = 5.740\times 10^{17}\mathrm{s}^{-1}$ and $h(z) := \sqrt{\pi}e^{z^2}\mathrm{erfc}(z)/2z$.

The relaxation time of the metastable with-field vacuum state via materialization is then the ratio of the rate of electromagnetic field energy conversion to the available supply of field energy density u_f. We define the time constant $\tau^{-1} := (d\langle u_m\rangle/dt)u_f^{-1}$. A rough time scale may be obtained by ignoring the second term Eq. (1) and the nonlinear corrections in the field energy, using $u_f^{(0)} = E^2/2$,

$$\tau^{(0)} := \frac{\omega_0^{-1}}{4}e^{\frac{\pi E_0}{E}}, \tag{2}$$

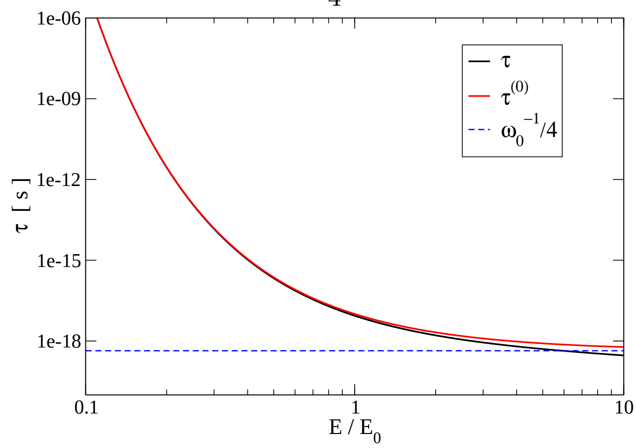

Fig. 1: Materialization time τ (solid line) and $\tau^{(0)}$ (lighter, red line) as a function of the external normalized field E/E_0. Dashed line: $\omega_0^{-1}/4 = \pi^2\hbar/4\alpha mc^2 = 0.435\,\mathrm{as}$.

In general, the longevity of the field depends on the real energy available to go into pairs, i.e. the proper energy density, $u_f = |v_\mu T^{\mu\nu}|$.

$d\langle u_m\rangle/dt$ is Lorentz invariant, so τ as defined is the Lorentz proper decay time of the vacuum. To add a constant, homogeneous magnetic field, we therefore need only consider the case $E\cdot B \neq 0$ and choose the frame in which E and B are either parallel or antiparallel. Transverse momentum states are now quantized, converting the momentum integral into a sum, from which we find

$$\frac{d\langle u_m\rangle}{dt} = \omega_0 E^2 e^{-\frac{\pi E_0}{E}}\left\{1 + h\left(\sqrt{\frac{\pi E_0}{E}}\right) - \chi(B^2, E)\right\} \tag{3}$$

$$\chi := \frac{E_0}{E}\sum_{k=1}^{\infty}\frac{\mathfrak{B}_{2k}}{(2k)!}\left(\frac{2B}{E_0}\right)^{2k} e^{\frac{\pi E_0}{E}}\frac{d^{2k-1}}{dx^{2k-1}}\left[\sqrt{x}e^{-\frac{\pi E_0}{E}x}\right]_{x=1}$$

with $\mathfrak{B}_{2k}$ the Bernoulli numbers. For $B \gtrsim E$ the lifetime of the field is increased over the pure electric case despite an augmented pair production rate.

From the example particle creation rates and field lifetimes displayed in Table 1, we see the percentage of the field energy converted generally remains small, so in practice materialization will not present a major source of field-energy dissipation. Yet, materialization of near-critical fields can lead to the formation of large $O(50\ \mathrm{nm}^3)$ spatial domains of electron-positron-photon plasma, probing the strongly coupled regime of QED and providing an accessible analogy for the current interest in quark-gluon plasma.

Table 1: Materialization characteristics (yield rate W, relaxation time τ) assuming complete materialization.

E/E_0	W [μm^{-3}fs^{-1}]	τ [fs]
0.0628	1	2.275×10^{18}
0.1	3.102×10^{8}	1.88×10^{10}
0.2	8.234×10^{15}	2800
0.402	9.68×10^{19}	1
1	5.903×10^{22}	8.85×10^{-3}

The assumption of adiabaticity is justifiable *a posteriori* for $E \to E_0$ seeing that characteristic times for materialization in critical fields are 10^3 times shorter than current typical intense pulse laser fields. On the other hand, the shorter field pulses generated in relativistic focusing require stronger fields to induce Schwinger tunneling, as distinct from frequency-aided, "induced" vacuum decay. However, the concept of materialization time can be generalized to more intense and shorter-lived field configurations within the same semiclassical approach.

References

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