Hadron Structure from Lattice QCD ◇

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Following successful lattice studies of form factors and moments of generalized parton distributions (GPDs) of the pion and the nucleon, in particular with respect to their transverse spin structures [1,2], we recently extended our efforts to the form factors of the spin-1 ρ-meson [3] in dynamical lattice QCD. The calculations are based on two flavors of improved Wilson fermions, for pion masses in the range of ≈ 350 MeV to ≈ 1000 MeV. Figure 1 displays exemplary results for the electromagnetic form factors of the ρ-meson as functions of the squared momentum transfer Q^2, for $m_\pi \approx 560$ MeV, together with mono-, dipole and linear fits represented by the shaded bands.

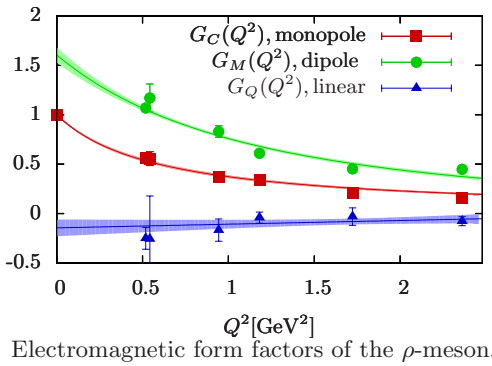

Figure 1: Electromagnetic form factors of the ρ-meson.

We observe good statistics for the charge, $G_C(Q^2)$, and the magnetic, $G_M(Q^2)$, form factors. While the central values for the quadrupole form factor $G_Q(Q^2)$ tend to be negative, most data points are compatible with zero within errors. From the mono- ($p = 1$) and dipole ($p = 2$) fits with pole masses m_p and the forward values as free parameters, we extract mean square radii and the magnetic moment, $\langle r^2\rangle = 6p/m_p^2$ and $\mu_\rho = G_M(0)$ (in natural units). The quadrupole moment, $Q_\rho = G_Q(0)/m_\rho^2$, is obtained from a linear extrapolation in Q^2. At a pion mass of ≈ 400 MeV closely above the ρ-to-$\pi\pi$ decay threshold, we find $\langle r^2\rangle_C = 0.45(2)$ fm^2 for the mean square charge radius, $\mu_\rho = 1.7(1)$, and $Q_\rho = -5.8(1.2)\,10^{-3}$ fm^2. A negative value for Q_ρ indicates an oblate shape of the ρ-meson, which in our case still has to be confirmed by improving the statistics and systematics, e.g. by employing partially twisted boundary conditions to access the region of very small non-zero Q^2 in Fig. 1. Similar results for the axial vector form factors $\tilde{G}_1(Q^2)$ and $\tilde{G}_2(Q^2)$ indicate that the total contribution of the quark spin to the spin $S = 1$ of the ρ^+-meson is $\tilde{G}^{u+d}_{1,\rho^+}(0)/2 = \Delta\Sigma/2 \approx 64\%$, comparable to the case of the nucleon, at pion masses of ≈ 400 MeV.

Important information on hadron structure, complementary to form factors and GPDs, is provided by transverse momentum dependent parton distributions (tmdPDFs), $f(x, k_\perp)$, which depend on the longitudinal momentum fraction x and the intrinsic transverse momentum $k_\perp$ of partons in the hadron. TmdPDFs play a central role in semi-inclusive deep-inelastic scattering and related azimuthal asymmetries, and, similarly to GPDs, have in general a probabilistic interpretation. They can be accessed through hadron matrix elements of non-local quark operators, $\langle P|\mathcal{O}(l)|P\rangle$, parametrized by invariant complex amplitudes $\tilde{A}_i(l^2, l\cdot P)$. Similarly to our previous study of $\tilde{A}_2$, we have recently performed a first lattice calculation of $\tilde{A}_7$ [4], which is related to distributions of longitudinally polarized quarks in a transversely polarized nucleon. Our investigations are based on a hybrid approach with $n_f = 2+1$ domain wall valence and staggered sea quarks. Substantial effort went into a proper renormalization of the relevant lattice operators.

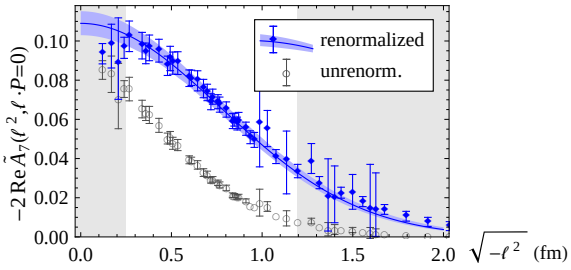

Figure 2: Real part of the amplitude $\tilde{A}_7$ for $(u-d)$-quarks.

The $|l|$-dependence of the renormalized amplitude $\tilde{A}_7$ at $l\cdot P = 0$ is displayed in Fig. 2 together with a Gaussian parametrization represented by the shaded band, for a pion mass of ≈ 500 MeV. By a Fourier-transformation, $\int d^2l_\perp \exp(il_\perp\cdot k_\perp)$, to $k_\perp$-space, the amplitudes $\mathrm{Re}\tilde{A}_2$ and $\mathrm{Re}\tilde{A}_7$ give direct access to transverse momentum dependent probability densities of quarks in the nucleon.

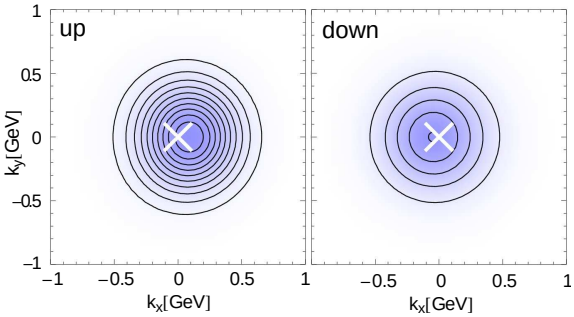

Figure 3: Transverse momentum densities of longitudinally polarized u- and d-quarks in a transversely polarized proton.

As a consequence of the clearly non-zero, even quite sizeable results for $\mathrm{Re}\tilde{A}_7$, cf. Fig. 2, we find that densities for quarks with helicity $\lambda = +1$ in a transversely polarized nucleon with spin $S_\perp = (1, 0)$ are visibly deformed due to dipole-correlations $\propto \lambda\, k_\perp \cdot S_\perp$, as shown in Fig. 3.

References

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