3.3 Atomic Physics and Astrophysics

3.3.1 Bound states of the two-center Dirac equation in a diatomic basis

W. Pöschl, M. Schaden, K. Dietrich

A good knowledge of the electron wavefunction near the nucleus is important for an accurate prediction of monoenergetic internal pair conversion (MPC) and internal conversion (IC) probabilities and relativistic effects become important in such calculations for $Z > 40$ nuclei [1]. Such relativistic effects on IC and MPC processes could even be more pronounced in the aftermath of heavy ion collisions near the coulomb barrier, when there is a possibility, that the electron of the final (initial) state is strongly bound in a molecular orbit of the combined two ion system. The common molecular orbit of the two ions in such systems would also allow for the unique possibility of observing correlated MPC and IC processes of the two ions [2].

The relative velocity $v/c \sim 0.1$ of the two ions in such collisions is small compared to mean electron velocities and a semiclassical treatment possible. The typical lifetimes of ordinary MPC and IC decays in heavy ions of the order of 10^{-14}sec would imply that these transitions occur mainly when the two ions are well separated even on the atomic scale and correlations are negligible. However, the transition probability for these processes depends exponentially on the available energy and the presence of the strongly bound 1σ molecular level formed in heavy ion collisions could reduce these lifetimes by a few orders of magnitude. But we expect that they remain large in comparison to the collision time of only $\simeq 10^{-21}$sec of the ions. To estimate these probabilities we thus would like to have reasonably good wavefunctions and energies for the situation where the two ions still have a common lowest molecular level but are already well separated.

We were thus led to consider the combined two-ion system and had to approximately solve the two-center Dirac-equation

$$\left(\vec{\alpha}\vec{p} + \beta m - \frac{Z_1\alpha^2}{|\vec{r}-\vec{R}_1|} - \frac{Z_2\alpha^2}{|\vec{r}-\vec{R}_2|}\right)\Psi(\vec{r}) = E\Psi(\vec{r})\,, \tag{1}$$

for values of the distance $R = |\vec{R}_1 - \vec{R}_2|$ of the two nuclei ranging from a few hundred to several thousand Fermi.

The (exact) numerical solution of (1) by the authors of Ref. [3] gives Dirac wavefunctions that are rather difficult to use in subsequent calculations. To obtain wavefunctions which are reasonably good near each nucleus at relatively large ion separation (≥ 200fm), an expansion

$$\Psi(\vec{r}) = \sum_i a_i\phi_{1i}(\vec{r} - \vec{R}_1) + \sum_i b_i\phi_{2i}(\vec{r} - \vec{R}_2), \tag{2}$$

in the diatomic basis of Dirac wavefunctions ϕ of nucleus 1 and 2, would be more practical for most applications. Such wavefunctions are by construction better near either nucleus than those that can be obtained from a multipole expansion of the potential to any finite (low) order. They correctly include the relativistic enhancement of the charge density near either nucleus that is essential for a reasonably good estimate of IC and MPC.

In contrast to the nonrelativistic case however, the Dirac Hamiltonian is not bounded below and there is no guarantee that the energy of the lowest molecular level is approached from above.

Nevertheless we found that the negative energy continuum has little influence at separations of the ions larger than about 200fm even in such highly charged systems as U+U. The largest diatomic basis we considered, using only the $\{\{1s, 2p_{1/2}, 2s\}_{1,2}\}$ atomic states of the two ions, reproduces the binding energy of the lowest molecular (1σ) level in the U+U system of the (exact) calculation by Ref.[3] beyond 500fm ion separation and is still good to about 10% down to 200fm. At distances below 200fm, when it becomes energetically favourable to occupy the 1σ-level, the influence of the negative energy continuum can probably no longer be neglected. The expansion of the wave function in a diatomic basis then loses its simplicity and appeal. Down to such small ion separations this expansion however seems to work and is rather simple to use, since only a few expansion coefficients (six for the above-mentioned basis) and the energy eigenvalue have to be determined numerically.

[1] R. Anholt, D.H. Jakubassa-Amundsen, P.A. Amundsen, K. Aashamar, Phys. Rev. **A27** (1983) 680.
[2] For a review of corresponding heavy ion experiments done at GSI see the Habilitationsschrift of H.Bokemeyer GSI-90-11
[3] B. Müller, W. Greiner, Zeit.f.Naturforsch. **31a** (1976) 1

3.3.2 Strange stars

N.K. Glendenning [a] and F. Weber
[a] Lawrence Berkeley Laboratory, Berkeley, California

The hypothesis that strange quark matter may be the absolute ground state of the strong interaction (not ^{56}Fe) has been raised by Witten in 1984. If the hypothesis is true, then a *separate* class of compact stars could exist, which are called strange stars. They form a distinct and disconnected branch of compact stars and are not part of the continuum of equilibrium configurations that include white dwarfs and neutron stars. In principle both strange and neutron stars could exist. However if strange stars exist, the galaxy is likely to be contaminated by strange quark nuggets which would convert all neutron stars that they come into contact with to strange stars. This in turn means that the objects known to astronomers as pulsars are probably rotating strange matter stars, *not* neutron matter stars as is usually assumed. Unfortunately the bulk properties of models of neutron and strange stars of masses that are typical for neutron stars, $1.1 \lesssim M/M^{\odot} \lesssim 1.8$, are relatively similar and therefore do not allow the distinction between the two possible pictures. The situation changes however as regards the possibility of *fast rotation* of strange stars. This has its origin in the completely different mass-radius relations of neutron and strange stars (see Fig. 1). As a consequence of this the entire family of strange stars can rotate rapidly, not just those near the limit of gravitational collapse to a black hole as is the case for neutron stars. As an example, above the minimum possible rotational periods of maximum-mass neutron and hybrid stars have been determined to be larger than ≈ 0.8 ms; this is to be compared with $\approx (0.4\text{-}0.6)$ ms calculated for maximum-mass strange stars [1, 2].

At the present time there appears to be only one crucial *astrophysical test* of the strange-quark-matter hypothesis, and that is whether strange quark stars can give rise to the observed phenomena of pulsar glitches. In the crust quake model an oblate solid nuclear crust in its present shape slowly comes out of equilibrium with the forces acting on it as the rotational period changes, and fractures when the built up stress exceeds the sheer strength of the crust material. The period and rate of change of period slowly heal to the trend preceding the glitch as the coupling between crust and core re-establish their co-rotation. the existence of glitches may have a decisive impact on the question of whether strange matter is the ground state of the strong interaction.

The only existing investigation which deals with the calculation of the thickness, mass and moment of inertia of the nuclear solid crust that can exist on the surface of a rotating, general relativistic strange quark star has only recently been performed by Glendenning and Weber [2]. Their calculated mass-radius relationship for strange stars with a nuclear crust, whose maximum density is the neutron drip density, is shown in Fig. 1. (Free neutron in the star cannot exist. These would be dissolved into quark matter as they gravitate into the core. Therefore the maximum density of the crust is strictly limited by neutron drip. This density is about 4.3 x 10^{11} g/cm^3.) Since the crust is bound by the gravitational interaction (and not by confinement, which is the case for the strange matter core), the relationship is qualitatively similar to the one for neutron and hybrid stars [1]. The radius being largest for the lightest and smallest for the heaviest stars (indicated by the solid dot in Fig. 1) in the sequence. Just as for neutron stars the relationship is not necessarily monotonic at intermediate masses. The radius of the strange quark core, denoted R_{drip}, is shown by the dashed line. (A value for the bag constant of $B^{1/4} = 160$ MeV for which 3-flavor strange matter is stable has been chosen. This choice represents weakly bound strange matter with an energy per baryon $\sim$ 920 MeV, and thus corresponds to strange quark matter being absolutely bound with respect to ^{56}Fe). The radius of the strange quark core is proportional to $M^{1/3}$ which is typical for self-bound objects. This proportionality is only modified near the mass where gravity terminates the stable sequence. The sequence of stars has a minimum mass of $\sim 0.015 M_\odot$ (radius of $\sim$ 400 km) or about 15 Jupiter masses, which is smaller than that of neutron star sequences, about 0.1 $M_\odot$ [1]. The low-mass strange stars may be of considerable importance since they may be difficult to detect and therefore may effectively *hide baryonic matter*. Furthermore, of interest to the subject of cooling of strange stars is the crust thickness of strange stars. It ranges from $\sim$ 400 km for stars at the lower mass limit to $\sim$ 12 km for stars of mass $\sim$ 0.02 $M_\odot$, and is a fraction of a kilometer for the star at the maximum mass [2].

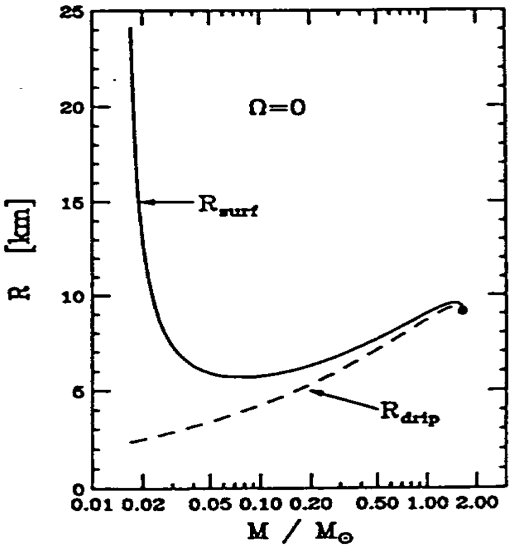

Fig. 1: Radius as a function of mass of a non-rotating strange star with crust [2].

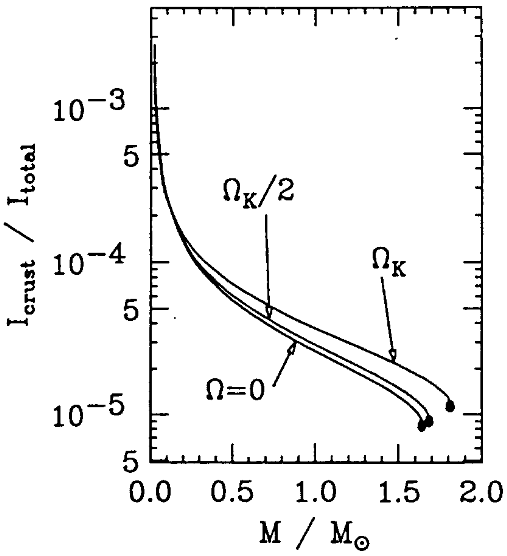

Fig. 2: The ratio I_{crust}/I_{total} as a function of star mass. Rotational frequencies are shown as a fraction of the Kepler frequency, Ω_K.

The moment of inertia of the hadronic crust, I_{crust}, that can be carried by a strange star as a function of star mass for a sample of rotational frequencies of $\Omega = \Omega_K, \Omega_K/2$ and 0 is shown in Fig. 2.

Because of the relatively small crust mass of the maximum-mass models of each sequence, the ratio I_{crust}/I_{total} is smallest for them (solid dots in Fig. 2). The less massive the strange star the larger its radius (Fig. 1) and therefore the larger both I_{crust} as well as I_{total}. The dependence of I_{crust} and I_{total} on M is such that their ratio I_{crust}/I_{total} is a monotonically decreasing function of M. One sees that there is only a slight difference between I_{crust} for $\Omega = 0$ and $\Omega = \Omega_K/2$.

Of considerable relevance for the question of whether strange stars can exhibit glitches in rotation frequency, one sees that I_{crust}/I_{total} varies between 10^{-3} and $\sim 10^{-5}$ at the maximum mass. If the angular momentum of the pulsar is conserved in the quake then the relative frequency change and moment of inertia change are equal, and one arrives at [2].

$$\frac{\Delta\Omega}{\Omega} = \frac{\mid \Delta I \mid}{I_0} > \frac{\mid \Delta I \mid}{I} \equiv f\frac{I_{crust}}{I} \sim (10^{-5} - 10^{-3})f \quad , \ with 0 < f < 1 \ . \tag{27}$$

Here I_0 denotes the moment of inertia of that part of the star whose frequency is changed in the quake. It might be that of the crust only, or some fraction, or all of the star. The factor f in Eq.(1) represents the fraction of the crustal moment of inertia that is altered in the quake, i.e. $\mid \Delta I \mid = f I_{crust}$. Since the observed glitches have relative frequency changes $\Delta\Omega/\Omega = (10^{-9} - 10^{-6})$, a change in the crustal moment of inertia by less than 10% would cause a giant glitch even in the least favorable case (for more details see Ref.[2]. Finally we find that the observed range of the fractional change in $\dot{\Omega}$ is consistent with the crust having the small moment of inertia calculated and the quake involving only a small fraction f of that, just as in Eq.(1). For this purpose we write [2]

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} = \frac{\Delta\dot{\Omega}/\dot{\Omega}}{\Delta\Omega/\Omega}\frac{\mid \Delta I \mid}{I_0} = \frac{\Delta\dot{\Omega}/\dot{\Omega}}{\Delta\Omega/\Omega} f \frac{I_{crust}}{I_0} > (10^{-1} to 10)f \quad , \tag{28}$$

which yields a small f value as before: $f < (10^{-4} to 10^{-1})$. Here measured values of the ratio $(\Delta\Omega/\Omega)/(\Delta\dot{\Omega}/\dot{\Omega}) \sim 10^{-6}$ to 10^{-4} for the Crab and Vela pulsars, respectively, have been used.

[1] For more details see: F. Weber and N.K. Glendenning: Neutron Stars, Strange Stars, and the Nuclear Equation of State; First Symposium on Nuclear Physics in the Universe; Oak Ridge, Tennessee, USA, to appear in The Journal of Physics (LBL-Report 33066

[2] N.G. Glendenning and F. Weber; Nuclear Solid crust on Rotating Strange Stars (LBL-Report 31613; to appear in The Astrophys. Journal)

3.3.3 Hadron physics and neutron star properties

F. Weber

Models of rotating neutron stars are constructed in the framework of Einstein's theory of general relativity. The properties of neutron stars, e.g. gravitational mass, equatorial and polar radius, red- and blueshift, eccentricity, quadrupole moment, are calculated for sequences of star models rotating at their respective general relativistic Kepler frequencies, Ω_K. The role of the gravitational radiation reaction-driven instability in rotating neutron stars is investigated too, and the corresponding rotational instability frequencies are calculated and compared with Ω_K.

The investigation is based on four neutron star matter (electrically charge neutral many-baryon/lepton system) equations of state determined in the framework of the relativistic Green's function method, as well as a representative collection of thirteen additional equations of state covering both non-relativistic as well as relativistic ones. The relativistic Hartree, Hartree-Fock, and ladder (i.e. relativistic

Brueckner-Hartee-Fock type) approximations are introduced to derive the models for the equation of state of neutron star matter in the relativistic Green's function approach. The characteristic feature of the ladder approximation consists in the inclusion of dynamical two-particle correlations in matter in terms of a relativistic (Bethe-Salpeter type) scattering T-matrix. For its determination both the HEA and Bonn meson-exchange potentials of the nucleon-nucleon force served as an input.

The main intentions of this work consist in (i) testing the compatibility of the behavior of different models for the nuclear equation of state at super-nuclear densities with data of pulsars (e.g. double constraint of fast rotation and a large enough neutron star mass), and (ii) the determination of the smallest possible rotational pulsar period obtainable from baryonic matter equations of state. By means of item (i) the fundamental problem of nuclear physics concerning the behavior of the equation of state at super-nuclear densities can be treated. Item (ii) may have far-reaching consequences for the interpretation of the possible future detection of *fast* pulsars.

In short, we find that in order to satisfy the double constraint of fast rotation and a large enough star mass, the nuclear equation of state needs to behave *soft at intermediate nuclear densities and considerably stiffer in the high-density region.* Furthermore, *none of the selected representative nuclear equations of state allows for neutron star rotation at periods considerably below* $0.8 - 0.9$ *ms.* Thus, this work strongly supports the suggestion that if pulsars with shorter rotational periods were found, these are likely to be *strange-quark-matter stars.* The conclusion that the confined hadronic phase of nucleons and nuclei is only *metastable* would then be almost inescapable, and the plausible ground-state in that event is the deconfined phase of *(3-flavor) strange-quark-matter!*

Weber, Habilitation Thesis LMU, 1992

3.3.4 Neutron stars, strange stars and the nuclear equation of state

F. Weber and N.K. Glendenning [a]
[a] Lawrence Berkeley Laboratory, Berkeley, California

This report consists of three parts. In part one, we review the present status of dense nuclear matter calculations, and introduce a representative collection of realistic nuclear equations of state which are derived for different assumptions about the physical behavior of dense matter (baryon population, pion condensation, possible transition of baryon matter to quark matter). In part two we review recently performed non-rotating and rotating compact star calculations performed for these equations of state. The minimum stable rotational periods of compact stars, whose knowledge is of decisive importance for the interpretation of rapidly rotating pulsars, are determined. In part three of this paper we investigate the amount of nuclear solid crust that can be carried by a rotating strange star, and answer the question whether such objects can give rise to the observed phenomena of pulsar glitches, which is at the present time the only astrophysical test of the strange-quark-matter hypothesis.

The indication of this work is that the gravitational radiation-reaction instability sets a lower limit on stable rotation for massive *neutron or hybrid stars* of $P \approx 0.8$ ms. Lighter ones having typical pulsar masses of $1.45\,M_\odot$ are predicted to have rotational periods $P \gtrsim 1$ ms. This finding may have very important implications for the nature of any pulsar that is found to have a shorter period, say below $P \lesssim 0.5$ ms. Since our representative collection of nuclear equation of state does not allow for rotation at such small periods, the interpretation of such objects as rapidly rotating neutron or hybrid stars fails. Such objects, however, can be understood as rapidly rotating *strange stars*. The plausible ground-state state in that event is the deconfined phase of (3-flavor) strange-quark-matter.

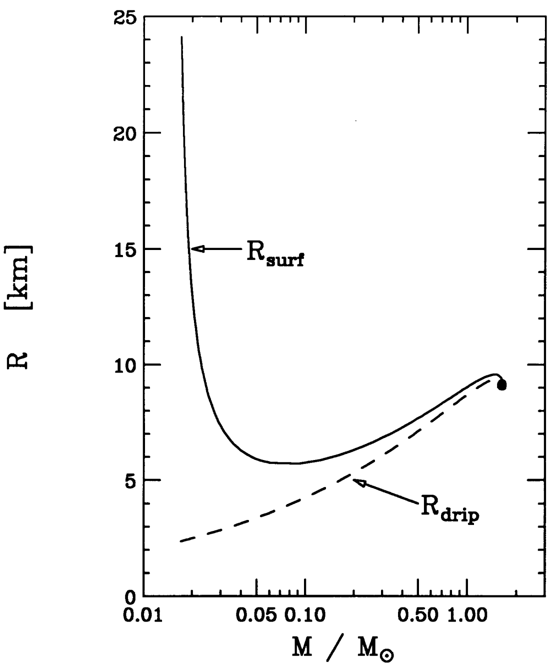

Fig. 1:

We find that the nuclear solid crust that can exist on the surface of a strange star (see between $R_{\rm drip}$ and $R_{\rm surf}$ in Fig. 1 of contribution Strange Stars by NKG and FW) can have a moment of inertia sufficiently large that a fractional change can account for the magnitude of pulsar glitches. Furthermore low-mass strange stars can have enormously large nuclear crusts (up to ~ 400 km) which might considerably alter the cooling rate of strange stars and enables such objects to be possible hiding places of baryonic matter.

F. Weber and N.K. Glendenning, "Interpretation of Rapidly Rotating Pulsars", presented at the Second International Symposium on Nuclear Astrophysics NUCLEI IN THE COSMOS, Karlsruhe, Germany, July 6-10, 1992, (LBL-32562); F. Weber and N.K. Glendenning, "Neutron Stars, Strange Stars, and the Nuclear Equation of State", presented at the First Symposium on Nuclear Physics in the Universe, Oak Ridge, TN, September 24-26, 1992, (LBL-33066)